**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Ans.**

Mean (**μ**) = 45 minutes

Std (**σ** ) = 8 minutes

**X**= 60-10 =50 minutes

The work begins after 10 min, so the average time increase from

45min to 55min.

for normal distribution: - X ~N(45,8)

Z = (X-μ)/σ

= (50-45)/8

= 0.625

Probability that the service manager will not meet his demand will be

P(X>50) = P ( (X-μ)/σ > (50-45)/8 )

= P( Z > 0.625 )

= 0.2676 [ 1 – stats.norm.cdf(abs(Z\_Score)) ]

OR

In R Software

(1 -pnorm(50,45,8)) = 0.2659

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**False**

Around 70% of the data falls within one standard deviation of the mean (µ+σ= 38+6=44)

If more employees are older than 44, this will shift the*μ*  towards 44 with considering sd. Which is not possible as *μ* is given 38 with Sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

Z=(X-µ)/ σ

Employees under 30 yrs of age P(X≤30) :

P(X≤30) =p (Z ≤ (30-38) /6)

= p (Z ≤ -1.33)

= 0.0918( by z table)

OR

Employees under 30 yrs of age P(X≤30) :

stats.norm.cdf (30,38,6)

Expected count=0.0918\*400= **36.72**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans.**

if X ∼ N (µ1, σ1^2) **And** Y ∼ N (µ2, σ2^2) are two independent random variables

then

X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2) **And** X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2)

Similarly, if Z = aX + bY i.e Z is linear combination of X and Y,

then

Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2)

Therefore

2X1~ N (2 u,4 σ^2)

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans. Option D**

The Probability of getting value between a and b is 0.99

Probability outside the a and b area is 0.01 (i.e. 1- 0.99).

0.01/2 =0.005

The Probability towards left from a = -0.005

The Probability towards Right from b = +0.005

For Probability 0.005 the Z Value is -2.57 (by Z Table)

Z =(X- μ) / σ

X = Z \* σ + μ

Z(-0.005)\*20+100 = **151.4**

Z(+0.005)\*20+100 = **48.6**

OR

stats**.**norm**.**interval(0.99,100,20)

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. **48.5, 151.5**
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans.**

Profit1~N(5, 32)

> qnorm(0.025,45\*5,3)

[1] 219.1201

> qnorm(0.975,45\*5,3)

[1] 230.8799

Profit2~N(7, 42)

> qnorm(0.025,45\*7,4)

[1] 307.1601

> qnorm(0.975,45\*7,4)

[1] 322.8399

95% probability for the annual profit of the company is,

=Profit1 + Profit2

=[219.12,230.87]+[307.16,322.83]

=[526.28,553.7]

So Profit of the company in Rupees(in **Million**) is between range **[528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans.**

> qnorm(0.05,45\*7,4)

[1] 308.4206

> qnorm(0.05,45\*5,3)

[1] 220.0654

5th percentile of profit = 308.4206+ 220.0654 = **528.486 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.**  Division 2 with distribution N(7, 42) has a larger probability of making a loss in

a given year